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1989 J. Phys. A: Math. Gen. 22 3629

(http://iopscience.iop.org/0305-4470/22/17/027)

J. Phys. A: Math. Gen. 22 (1989) 3629-3636. Printed in the UK

Low-field magneto-resistance in a regular fractal model

Hanan Rosenthal and David J Bergman

School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel

Received 22 March 1989

Abstract. We calculate the scaling behaviour of the magneto-resistance of a regular three-dimensional (3D) fractal network model in a weak magnetic field. The method used is a discretised version of a previously described continuum theory. Exponents are calculated that characterise the power-law dependence of both the transverse and the longitudinal magneto-resistance on the total size. Results are compared with a previously published prediction of scaling theory for the magneto-resistance near the percolation threshold.

The Hall effect has been used extensively to investigate the metal–non-metal transition in a variety of disordered systems. An effective medium theory (EMT, Cohen and Jortner 1973, Stroud and Pan 1979, Stachowiak 1970), scaling theories (Shklovskii 1977, Bergman 1983 and 1987, Bergman and Stroud 1985) and a simulation approach (Webman *et al* 1975, Bergman *et al* 1983) have been used to discuss the properties of the Hall effect in conductors with macroscopic disorder, i.e. composite conductors. Straley (1980) generalised the Cayley tree model to also apply to the Hall effect. The case of a two-dimensional, macroscopically inhomogeneous system, with or without disorder, was discussed in a number of articles (Juretschke *et al* 1956, Straley 1980, Bergman 1983, Stroud and Bergman 1984), with the conclusion that the results for the bulk effective magnetotransport coefficients are exactly expressible in terms of the bulk properties at zero magnetic field.

Bergman (1987) has reported on a new exact theory for the magneto-resistance ($\delta\rho$) of a two-component isotropic composite and used it to construct a scaling theory for $\delta\rho$ of random composites. He showed that in the case of composite conductors in a low magnetic field, the effective Ohmic conductivity (σ_e), Hall conductivity (λ_e) and the magneto-conductivity ($\delta\sigma_e$), can be obtained without having to solve for the local electric potential $\phi(\boldsymbol{r})$ in the presence of a magnetic field $\boldsymbol{H}$. The expressions describing the different conductivities in the continuum case are ($\boldsymbol{H}$ parallel to z):

$$\frac{\sigma_e-\sigma_I}{\sigma_M-\sigma_I}=\frac{1}{V}\int \mathrm{d}V\,\theta_M(\boldsymbol{r})\left(\frac{\partial\phi^{(ox)}}{\partial x}\right) \tag{1}$$

$$\frac{\lambda_e-\lambda_I}{\lambda_M-\lambda_I}=\frac{1}{V}\int \mathrm{d}V\,\theta_M(\boldsymbol{r})(\nabla\phi^{(ox)}\times\nabla\phi^{(oy)})_z \tag{2}$$

$$\boldsymbol{a}\delta\sigma_e\boldsymbol{b}=\frac{1}{V}\int \mathrm{d}V\nabla\phi^{(oa)}\delta\sigma(\boldsymbol{r})\nabla\phi^{(ob)}+\frac{(\lambda_M-\lambda_I)^2}{V}\int \mathrm{d}V\,\theta_M(\boldsymbol{r})\int \mathrm{d}V'\,\theta_M(\boldsymbol{r}')\times(\hat{\boldsymbol{H}}\times\nabla\phi^{(oa)})(\hat{\boldsymbol{H}}\times\nabla'\phi'^{(ob)}):\nabla\nabla' G. \tag{3}$$

Here, $\phi^{(of)}$ is the local electric potential at zero magnetic field when a voltage drop is applied to the system, that would result in a uniform electric field equal to the unit vector $\boldsymbol{f}$ if the system were homogeneous. Also, σ_M, σ_I and λ_M, λ_I are the Ohmic and Hall conductivities of the two components, and $\delta\sigma(\boldsymbol{r})$ is the local magneto-conductivity tensor (second order in the magnetic field). The step function $\theta_M(\boldsymbol{r})$ equals 1 if $\boldsymbol{r}\epsilon\sigma_M$ and zero otherwise. In these expressions all the integrals are confined to the σ_M volume, except for the first integral of (3). The function $G(\boldsymbol{r}, \boldsymbol{r}')$ is the Green function for the Ohmic ($H=0$) problem in the composite, $\phi'^{(ob)}$ is shorthand for $\phi(\boldsymbol{r}')^{(ob)}$ and $\hat{\boldsymbol{H}}$ is a unit vector along the direction of $\boldsymbol{H}$. Finally, $\boldsymbol{a}$ and $\boldsymbol{b}$ are two arbitrary unit vectors.

In order to determine the critical behaviour of λ_e of an isotropic composite, Bergman *et al* (1983) realised the Hall problem on a two-component discrete lattice as follows: each element of the lattice is a triplet of identical conductors, with an Ohmic conductance σ_I or σ_M, that lie along the coordinate axes, and which are electrically unconnected in the absence of a magnetic field $\boldsymbol{H}$ (figure 1(*a*)). In the presence of $\boldsymbol{H}$ along, say, the z axis, a Hall current will flow through a conductor in the x direction that is equal to the product of its Hall conductance (λ_I or λ_M) and the voltage across the y conductor of the same triplet. Two types of triplets are placed randomly at all the sites of a FCC lattice, and electrical connections are made at the centres of all the unit cell edges as well as the body-centre points (figure 1(*b*)). By making these connections, one obtains four simple cubic random resistor networks that are electrically unconnected but are strongly correlated with each other. Although the appearance of a number of unconnected networks is unphysical, this model nevertheless yields the correct critical behaviour for λ_e as well as for $\delta\sigma_e$ in two dimensions (2D). Other simpler models fail in this respect (see Bergman 1983). We hope that the unphysical dichotomy of the networks into a number of unconnected pieces does not affect the critical behaviour in 3D systems as well — this was indeed recently found to be true at least in the case of the low-field Hall effect (Bergman *et al* 1989).

In order to imitate in a simple way the percolating cluster in such a lattice, we used a regular fractal model proposed by Nagatani (1986b). This fractal is a 3D version

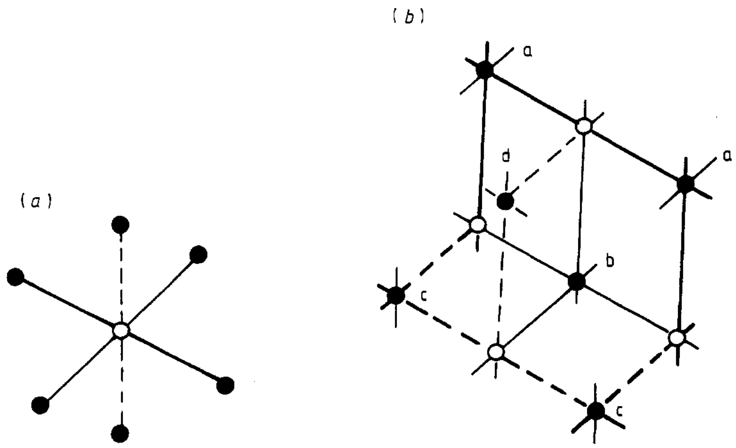

Figure 1. Schematic drawing of a portion of the random resistor network used to realise the Hall effect in a discrete system in 3D. (*a*) A triplet of identical, unconnected mutually perpendicular conductors. All networks are constructed from such unit elements. (*b*) A FCC lattice of elementary triplets. The centre of each elementary triplet is denoted by an open circle. Electrical connections are denoted by full circles. The 3D network is composed of four unconnected simple cubic resistor networks marked as *a*, *b*, *c*, *d*.

of a 2D variant of the Mandelbrot–Koch curve (Nagatani 1986a). It is shown in figure 2 along with its backbone, and has fractal dimensions that are very close to those of the percolating cluster and its backbone in a real random network at its percolation threshold. It consists of four unconnected networks, three of which are identical. Each of these is connected to the outside world by a pair of terminals lying along the x, y or z directions. The fourth network is an internal structure and has no connections to the outside. Thus, it does not affect either σ_e or λ_e but only $\delta\sigma_e$. Like the percolating cluster of a random network, this fractal includes three types of bonds, namely singly connected, multiply connected and dangling bonds. In order to perform calculations on this model, we need the discrete network version of (1)–(3), generalised so as to apply to systems with cubic rather than isotropic symmetry. In continuum materials with cubic symmetry, the current density up to $O(H^2)$ is given (see Seitz 1950) by

$$\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \left(\sigma \mathsf{I} + R_{\mathrm{H}}\sigma^2 \begin{bmatrix} 0 & H_z & -H_y \\ -H_z & 0 & H_x \\ H_y & -H_x & 0 \end{bmatrix} + \alpha H^2 \mathsf{I} \right.$$
$$\left. + \begin{bmatrix} (\beta+\gamma)H_x^2 & \beta H_x H_y & \beta H_x H_z \\ \beta H_y H_x & (\beta+\gamma)H_y^2 & \beta H_y H_z \\ \beta H_z H_x & \beta H_z H_y & (\beta+\gamma)H_z^2 \end{bmatrix} \right) \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}. \tag{4}$$

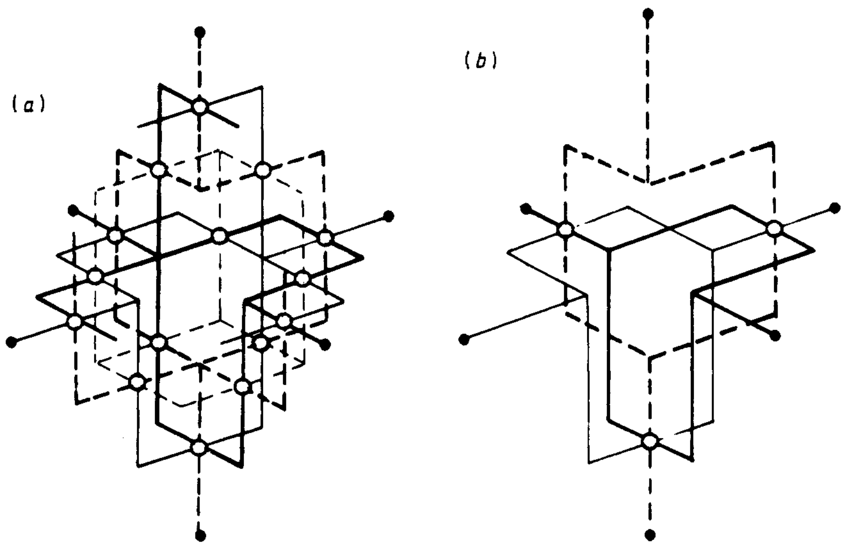

Figure 2. (a) The three-dimensional, $L=3$ fractal model used in our calculations of the weak-field magneto-conductance. This is the same as the fractal used by Nagatani (1986b) for calculations of the weak-field Hall effect. It is made of 15 elementary triplets, marked by open circles, that conspire to form four unconnected networks. Three of those (full lines, bold full lines and bold broken lines) are essentially identical though differently oriented in space, each one having two external terminals along one of the coordinate axes. The fourth network (broken lines) is different—it is entirely internal with no connections to the outside world. Nevertheless, it contributes to the magneto-conductance.

(b) The percolating backbone of (a). It is composed of three identical, electrically unconnected, networks.

Here, R_H is the Hall coefficient ($\boldsymbol{\lambda} = R_H\sigma^2 \boldsymbol{H}$), $\mathbf{I}$ is the unit matrix and (α, β, γ) are three independent coefficients which characterise the second-order terms in $\boldsymbol{H}$. Choosing $\boldsymbol{H}$ parellel to z, we can identify the longitudinal and transverse magneto-conductivities, respectively, as: $\delta\sigma_{\|} = (\alpha+\beta+\gamma)H^2$ and $\delta\sigma_{\perp} = \alpha H^2$. However, if we choose $\boldsymbol{H}$ parallel to $(\boldsymbol{x}+\boldsymbol{y}+\boldsymbol{z})$, then we find $\delta\sigma_{\|} = (\alpha+\beta+\frac{1}{3}\gamma)H^2$ and $\delta\sigma_{\perp} = (\alpha+\frac{1}{3}\gamma)H^2$. The fact that these differ from the previous expressions is due to the cubic symmetry. In an isotropic material, there are only two independent H^2 coefficients ($\gamma = 0$) and consequently $\delta\sigma_{\|}$ and $\delta\sigma_{\perp}$ do not depend on the direction of H†.

In our discrete lattice, where each bond lies along one of the coordinate axes; we assume the following form for the current through a bond a, by analogy with the continuum expression (4):

$$J_a^{(f)} = \sigma_I(1-\theta_a u)V_a^{(f)} + \lambda_I \sum_{\tilde{a}\in a}(1-\theta_a w)V_{\tilde{a}}^{(f)}(\hat{\boldsymbol{H}}, \boldsymbol{a}, \tilde{\boldsymbol{a}}) + (\alpha_a H^2 + \gamma_a H_a^2)V_a^{(f)} + \sum_{\tilde{a}\in a}\beta_a H_a H_{\tilde{a}} V_{\tilde{a}}^{(f)}. \tag{5}$$

Here $u \equiv (1-\sigma_M/\sigma_I)$, $w \equiv (1-\lambda_M/\lambda_I)$ and θ_a is a step function which is equal to 1 if a is a σ_M bond and equal to zero otherwise. The magnetic field component along bond a is H_a, and $(\hat{\boldsymbol{H}}, \boldsymbol{a}, \hat{\boldsymbol{a}}) \equiv \hat{\boldsymbol{H}} \cdot (\boldsymbol{a} \times \tilde{\boldsymbol{a}})$ is the triple product of the unit vectors $\hat{\boldsymbol{H}}, \boldsymbol{a}, \tilde{\boldsymbol{a}}$ lying along H, a, $\tilde{a}$. The relation $\tilde{a} \in a$ signifies that the two bonds belong to the same basic triplet. Thus, if $\boldsymbol{H}$ is also along a coordinate axis, there is only one non-zero term in the first sum, i.e. when a is perpendicular to both $\tilde{a}$ and H.

From this model one obtains the following expressions, analogous to (1)–(3) (see

$$\frac{\sigma_e - \sigma_I}{\sigma_M - \sigma_I} = \frac{1}{N}\sum_a \theta_a V_a^{(ox)} \tag{6}$$

$$\frac{\lambda_e - \lambda_I}{\lambda_M - \lambda_I}(\hat{\boldsymbol{H}}, \boldsymbol{e}, \boldsymbol{f}) = \frac{1}{N}\sum_{a,\tilde{a},\tilde{a}\in a} V_a^{(oe)}\theta_a V_{\tilde{a}}^{(of)}(\hat{\boldsymbol{H}}, \boldsymbol{a}, \tilde{\boldsymbol{a}}) \tag{7}$$

$$\boldsymbol{e}\delta\sigma_e\boldsymbol{f} = \frac{1}{N}\sum_a V_a^{(oe)}(\alpha_a + \gamma_a H_a^2)V_a^{(of)} + \frac{1}{N}\sum_{a,\tilde{a},\tilde{a}\in a} V_a^{(oe)}\beta_a H_a H_{\tilde{a}} V_{\tilde{a}}^{(of)} - \frac{(\lambda_M - \lambda_I)^2}{N}\sum_{\substack{a,\tilde{a},\tilde{a}\in a\\ b,\tilde{b},\tilde{b}\in a}} V_a^{(oe)}\theta_a \Gamma_{\tilde{a}\tilde{b}}\theta_b V_b^{(of)}(\hat{\boldsymbol{H}}, \tilde{\boldsymbol{b}}, \boldsymbol{b})(\hat{\boldsymbol{H}}, \boldsymbol{a}, \tilde{\boldsymbol{a}}). \tag{8}$$

Here, $V_a^{(oe)}$ is the voltage drop along bond a in the absence of a magnetic field when the average electric field is the unit vector $\boldsymbol{e}$. The symmetric matrix Γ_{ab} is the analogue of the tensor $\nabla\nabla' G$ and has the following physical significance: it is the voltage induced along the bond b when a current of unit strength is injected at one end and extracted at the other end of bond a. All this must be done while holding the boundary at a fixed potential.

In a translationally invariant system (e.g. a uniform lattice), summing Γ_{ab} over all bonds a yields zero (see Bergman and Kantor 1981). However, since our fractal does not possess this property, such a sum would usually not vanish. In (6)–(8) $N = L^3$ where L is the linear size of the sample. The first two terms on the right-hand side of (8) involve the three independent magneto-conductance coefficients of the unit

† The precise form of (4) can be obtained by expanding J_i as follows: $J_i = \sigma_{ij}E_j + \lambda_{ijk}H_jE_k + \eta_{ijkl}H_jH_kE_l + \ldots$, and analysing the form of η (which is similar to the elastic stiffness tensor C_{ijkl}) under the various point symmetries. It then follows that η has only two independent components for an isotropic material, and three such components for a cubic material.

element—α, β, γ. They are analogous to the first term on the RHS of (3). The last term of (8) is analogous to the last term of (3)—it is quadratic in the difference between the two Hall conductivities and arises when the first-order induced Hall currents produce a first-order correction to the local voltages, and these in turn produce a second-order Hall correction to the local currents.

In our model we assume that one of the components is a perfect insulator ($\sigma_I = \lambda_I = \delta\sigma_I = 0$), and therefore all properties are determined by clusters of the good conductor σ_M, λ_M and $\delta\sigma_M$. We use the regular fractal model described above to represent these clusters and we now have to find the bond voltages $V_a^{(oe)}$ and Green matrix Γ_{ab} for all bonds of our fractal. Note that although $V_a^{(oe)} \neq 0$ only for bonds of the backbone, $\Gamma_{ab} \neq 0$ also for dangling bonds and for bonds on the non-percolating internal network. Choosing appropriate sets of unit vectors $\boldsymbol{e}, \boldsymbol{f}$ and $\hat{\boldsymbol{H}}$, we calculate both $\delta\sigma_{e\parallel}$ and $\delta\sigma_{e\perp}$ in terms of σ_M, λ_M, α_M, β_M and γ_M.

Our purpose is to calculate $\delta\sigma_e$ both for the elementary ($L=1$) fractal of figure 1(a), and for the $L=3$ fractal of figure 2(a), in order to determine how it scales with the size L. Unfortunately, it turns out that the $(\lambda_M - \lambda_I)^2$ term of (8) vanishes for $L=1$ when we use Γ_{ab} as defined above, namely with fixed-potential boundary conditions. In order to permit some non-trivial estimate of the scaling behaviour of this term, we therefore used a different Γ_{ab}, defined by using zero-normal-current boundary conditions. In the limit of large L both types of Green matrices should lead to the same results since they will only differ appreciably near the surface.

The results for $\boldsymbol{e}\delta\sigma_e\boldsymbol{f}$ which depend on the choice of the three unit vectors $\boldsymbol{e}, \boldsymbol{f}, \hat{\boldsymbol{H}}$ were found to contain only three independent coefficients, so that the tensor $\delta\sigma_e$ for the network could be expressed by an equation like (4), but with bulk effective magneto-conductivity coefficients α_e, β_e, γ_e. This is not altogether surprising in view of the cubic structure of the fractal, although its point symmetry is lower than the full cubic symmetry group. From our calculations we find that the scaling behaviour with L is most simply expressed in terms of certain linear combinations of α_e, β_e and γ_e, namely

$$\begin{aligned}
(3\alpha_e+\beta_e+\gamma_e) &= (3\alpha_M+\beta_M+\gamma_M)L^{-\tilde{t}} + \frac{\lambda_M^{\,2}}{\sigma_M H^2} L^{-\tilde{t}_M} \\
(\beta_e+\gamma_e) &= (\beta_M+\gamma_M)L^{-\tilde{a}_3} - \frac{\lambda_M^2}{\sigma_M H^2} L^{-\tilde{a}_4} \\
\beta_e &= \beta_M L^{-\tilde{a}_1} - \frac{\lambda_M^2}{\sigma_M H^2} L^{-\tilde{a}_2}
\end{aligned} \tag{9}$$

where $\tilde{t} \equiv t/\nu$ is the critical exponent for the Ohmic conductivity, $\tilde{t}_M \equiv t_M/\nu$ is a new critical exponent defined by Bergman (1987) for the second-order contribution in the Hall effect (λ^2 term in (8)), ν is the percolation correlation length critical exponent, and a_1, a_2, a_3 and a_4 turned out to be corrections to the asymptotic scaling behaviour. In our model we get

$$\begin{aligned}
\tilde{t} &= 1 + \frac{\ln\frac{7}{2}}{\ln 3} \simeq 2.14 & \tilde{t}_M &= \frac{\ln\frac{245}{21}}{\ln 3} \simeq 2.24 \\
\tilde{a}_1 &= \frac{\ln\frac{49}{2}}{\ln 3} \simeq 2.91 & \tilde{a}_2 &= \frac{\ln\frac{105}{4}}{\ln 3} \simeq 2.97 \\
\tilde{a}_3 &= \frac{\ln\frac{147}{8}}{\ln 3} \simeq 2.65 & \tilde{a}_4 &= \frac{\ln\frac{245}{18}}{\ln 3} \simeq 2.38.
\end{aligned} \tag{10}$$

If we choose the magnetic field along, say, the z axis, and express the transverse and the longitudinal magneto-conductivities of the fractal in terms of those of the good conductor ($\delta\sigma_{M\perp}$ and $\delta\sigma_{M\parallel}$), then the leading terms, characterising the effective magneto-conductivities, are

$$\delta\sigma_{e\perp} = \delta\sigma_{e\parallel} \simeq (\tfrac{1}{3}\delta\sigma_{M\parallel} + \tfrac{2}{3}\delta\sigma_{M\perp})L^{-\tilde{t}} + \frac{1}{3}\frac{\lambda_M^2}{\sigma_M}L^{-\tilde{t}_M}. \tag{11}$$

Finally, since experiments directly measure the magneto-resistance $\delta\rho$ instead of $\delta\sigma$, we note that the Hall conductivity enters again when we express the relation between them ($\boldsymbol{H}$ parallel to z):

$$\begin{aligned} \frac{\rho_{e,xx}(\boldsymbol{H}) - \rho_{e,xx}(o)}{\rho_{e,xx}(o)} &\equiv \frac{\delta\rho_{e\perp}}{\rho_e} \simeq -\frac{\delta\sigma_{e\perp}}{\sigma_e} - \frac{\lambda_e^2}{\sigma_e^2} \\ \frac{\rho_{e,zz}(\boldsymbol{H}) - \rho_{e,zz}(o)}{\rho_{e,zz}(o)} &\equiv \frac{\delta\rho_{e\parallel}}{\rho_e} \simeq -\frac{\delta\sigma_{e\parallel}}{\sigma_e}. \end{aligned} \tag{12}$$

The asymptotic behaviour of $\delta\rho_e$ was thus found to be

$$\frac{\delta\rho_{e\perp}}{\rho_e} = \frac{\delta\rho_{e\parallel}}{\rho_e} \simeq \left(\frac{1}{3}\frac{\delta\rho_{M\parallel}}{\rho_M} + \frac{2}{3}\frac{\delta\rho_{M\perp}}{\rho_M}\right)L^0 + \tfrac{1}{3}\rho_M^2\lambda_M^2(2L^0 - L^{\tilde{t}-\tilde{t}_M}). \tag{13}$$

In (13), $\rho_M = 1/\sigma_M$ and $\delta\rho_M$ is the magneto-resistance of the good conductor.

Using finite-size scaling (Stauffer 1985), (11) and (13) can be written as functions of $\Delta p \equiv |p_M - p_c|$ (here p_M is the volume fraction of the good conductor and p_c is its percolation threshold) when $\Delta p \ll 1$ and $L = \infty$:

$$\begin{aligned} \delta\sigma_e &\simeq (\tfrac{1}{3}\delta\sigma_{M\parallel} + \tfrac{2}{3}\delta\sigma_{M\perp})\Delta p^{t} + \tfrac{1}{3}\frac{\lambda_M^2}{\sigma_M}\Delta p^{t_M} \\ \frac{\delta\rho_e}{\rho_e} &\simeq \left(\frac{1}{3}\frac{\delta\rho_{M\parallel}}{\rho_M} + \frac{2}{3}\frac{\delta\rho_{M\perp}}{\rho_M}\right)\Delta p^0 + \tfrac{1}{3}\rho_M^2\lambda_M^2(2\Delta p^0 - \Delta p^{t_M - t}). \end{aligned}$$

Equations (14) can be compared with a previously published prediction of a scaling theory (Bergman 1987). In that paper Bergman assumed that all the integrals (1)-(3) depend upon the same scaling variable and a scaling ansatz was written for the asymptotic form of (1)-(3) for the case $\sigma_M \gg \sigma_I$ and $\Delta p \ll 1$. In the case of a perfect insulator the same asymptotic form is obtained in our model with t as the leading exponent for the magneto-conductivity. Also, Bergman defined two new exponents, $t_{M\perp}$ and $t_{M\parallel}$, to characterise the behaviour of the λ_M^2 term of (3) for the transverse and longitudinal magneto-conductivities, respectively. In our model, we get $\tilde{t}_{M\perp} = \tilde{t}_{M\parallel} \equiv \tilde{t}_M \simeq 2.24$ (see equation (10)). We note that the first term on the RHS of (13) is a simple average of the longitudinal and transverse magneto-resistivities of the conducting component, with the ratio 1:2. This appears to result from the fact that the local electric field in the fractal is aligned with the various directions x, y, z with equal likelihood. We also note that even if $\delta\rho_{M\perp} = \delta\rho_{M\parallel} = 0$, as would be the case for a one-band, free-electron-gas conductor, our non-uniform fractal model will have both $\delta\rho_{e\perp} \neq 0$ and $\delta\rho_{e\parallel} \neq 0$ due to the second-order Hall effect (the second term of (13)). Also, in our model we always get $\delta\rho_{e\parallel} = \delta\rho_{e\perp}$, even if in the conducting component we had $\delta\rho_{M\perp} \neq \delta\rho_{M\parallel}$.

To summarise, we studied the scaling behaviour of the weak-field magneto-resistance on a regular fractal model. Second-order contributions in the magnetic field were included. We found that the effective magneto-conductivity of the fractal network can be characterised by three independent coefficients, which are related to the transverse and longitudinal magneto-conductivities, when $\boldsymbol{H}$ points in different directions. Exponents characterising the critical behaviour of the magnetotransport were calculated including, for the first time, the new critical exponent $\tilde{t}_M = 2.24$ which characterises the weak-field magneto-conductance.

The finite-size scaling exponents allow the values of a physical quantity at different length scales to be related to each other. This often results when a renormalisation group transformation is applicable that replaces a finite portion of the system by a new unit element. The possibility of doing this in the case of the Ohmic and the Hall conductivities is related to the fact that the sums in (6) and (7) can be separated rather easily into partial sums over different subsystems, apart from unimportant interaction terms at the interfaces. In the case of the magnetoconductivity, the sums in (8) cannot be separated in this way because the Green matrix Γ_{ab} is long ranged. Since it decreases only as the inverse cube of the distance between a and b, the double sum is highly non-local, and always includes pairs of bonds from different subsystems. For this reason, it was not possible to construct an exact real-space renormalisation group for the magnetoconductance even on the simple fractal model considered here.

We note also that Γ_{ab} can connect bonds that are not on the backbone or not even on a percolating cluster, e.g. the internal network. Therefore we had to consider all bonds of the fractal network, including those on dangling ends and on finite, non-percolating clusters.

We have reported here on the first calculation of scaling behaviour of the magneto-conductance in a 3D system. We hope that this will stimulate further efforts. Still needed is a calculation of scaling behaviour near a 3D percolation threshold as well as some careful experiments.

Acknowledgment

This research was supported in part by the US-Israel Binational Science Foundation under Grant No 354/85.

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